EECE 440
HW2 Solution

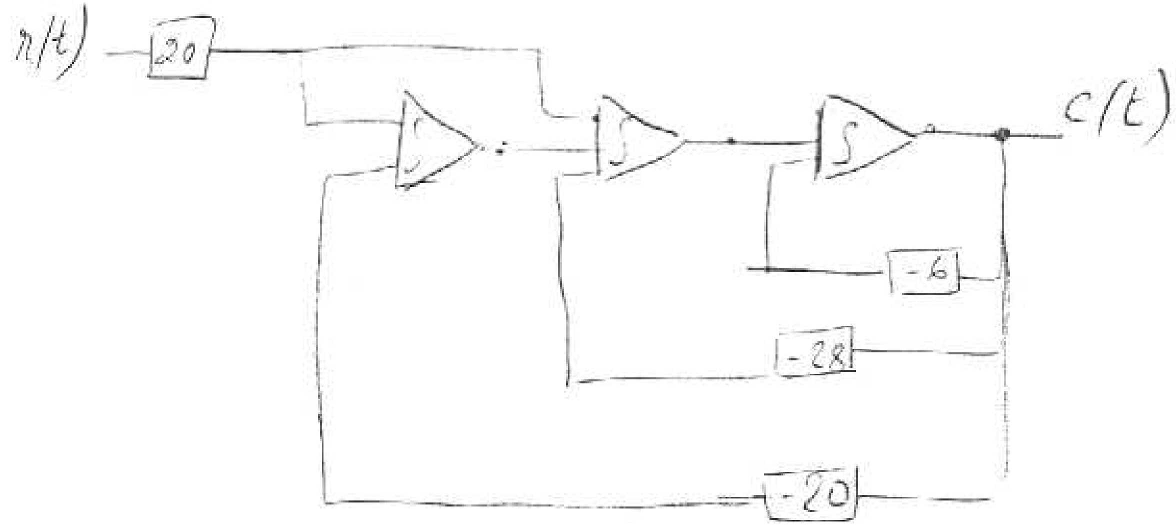
Part I

Problem 1

$$G(s) = \frac{20(s+1)}{s(s+2)(s+4)}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{20s+20}{s^3+6s^2+28s+20}$$

$$c(t) = -6\int c(t) - 28\iint c(t) - 20\iiint c(t) + 20\iint r(t) + 20\iiint r(t)$$



$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \end{bmatrix} = \begin{bmatrix} -6 & 1 & 0 \\ -28 & 0 & 1 \\ -20 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 20 \\ 20 \end{bmatrix} r(t)$$

$$c(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

Problem 2

$$A(s) = \frac{(s+1)}{2s^4 + 4s^3 + 4s^2 + as + 1}$$

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{s+1}{2s^4 + 4s^3 + 4s^2 + (a+1)s + 2}$$

$a = 0$

RH-table

(+) s^4	2	4	2
(+) s^3	4	1	
(+) s^2	$7/4$	1	
(-) s^1	$-9/7$		
(+) s^0	4		

$\Rightarrow$ 2 Sign Changes
$\Rightarrow$ System is NOT STABLE

$a = 8$ R-H Table

(+) s^4	2	4	2
(+) s^3	4	9	
(-) s^2	-1	4	
(+) s^1	25		
(+) s^0	4		

$\Rightarrow$ 2 Sign Changes
$\Rightarrow$ System is NOT STABLE

Problem 3

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} r(t) \qquad y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

a- $$\frac{Y(s)}{X(s)} = \frac{2s+5}{s^2+s-4} = \frac{2s+5}{\left(s+\frac{1}{2}-\frac{\sqrt{17}}{2}\right)\left(s+\frac{1}{2}+\frac{\sqrt{17}}{2}\right)}$$

$\Rightarrow$ pole in RHP

$\Rightarrow$ Unstable system

b. $$r(t) = \begin{bmatrix} f_1 & f_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} f_1 x_1 + f_2 x_2 \\ f_1 x_1 + f_2 x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -1+f_1 & 2+f_2 \\ 2+f_1 & f_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{DEN of TF} = \det[sI-A] = \begin{bmatrix} s+1-f_1 & -2-f_2 \\ -2-f_1 & s-f_2 \end{bmatrix}$$

$$= (s+1-f_1)(s-f_2) - (-2-f_1)(-2-f_2)$$

$$= s^2 - f_2 s + s - f_2 - f_1 s + f_1 f_2 - 4 - 2f_2 - 2f_1 - f_1 f_2$$

$$= s^2 + s\left[1-(f_1+f_2)\right] - 3f_2 - 2f_1 - 4$$

$$\text{Pole @ } -5 \Rightarrow \begin{cases} 25 - 5 + 5(f_1+f_2) - 3f_2 - 2f_1 - 4 = 0 \\ 9 - 3 + 3(f_1+f_2) - 3f_2 - 2f_1 - 4 = 0 \end{cases}$$
$$\text{Pole @ } -3 \Rightarrow$$

$$\Rightarrow f_1 = -2 \; ; \; f_2 = -5$$

Problem 4:

$$TF = \frac{2K(s+10)(s+1)}{s^3(2K+1) + s^2(12+22K) + s(20+40K) + 20K}$$

Oscillatory System when $D(j\omega) = 0$

$$-j\omega^3(2K+1) - \omega^2(12+22K) + j\omega(20+40K) + 20K = 0$$

$$\begin{cases} \omega(20+40K) - \omega^3(2K+1) = 0 \\ 20K - \omega^2(12+22K) = 0 \end{cases}$$

$\Rightarrow K = -\frac{1}{2}$ rejected because $\omega < 0$

or

$K = -\frac{4}{7} \Rightarrow \omega = 2\sqrt{5}$

2) $T = \frac{2\pi}{\omega} = \frac{2\pi}{2\sqrt{5}} = \frac{\pi}{\sqrt{5}}$

3) RH Table

			$-4/7$	$-6/11$	$-1/2$	0
s^3	$2K+1$	$-$	$-$	$-$	$+$	$+$
s^2	$11K+6$	$-$	$-$	$+$	$+$	$+$
s^1	$\frac{14K^2+15K+4}{11K+6}$	$-$	$+$	$-$	$+$	$+$
s^0	$10K$	$-$	$-$	$-$	$-$	$+$

No Sign Changes: Stable

$K \in]-\infty, -\frac{4}{7}[\cup]0, +\infty[$

4- Stable System

$$E_{ss} = \lim_{s \to 0} sE(s)$$

$$= \frac{C_{=}^{t} (s+10)(s+2)}{s^3(1+2k)+s^2(12+22k)+s(20+40k)+20K}$$

$$E_{ss} = \lim_{s \to 0} sE(s) = \frac{0}{20K} = 0$$

Problem 5:

$$\frac{C(s)}{R(s)} = \frac{K}{s+1}$$

a- For $K=10$, $r(t) = u(t) \Rightarrow C(s) = \frac{10}{s(s+1)} = \frac{10}{s} - \frac{10}{s+1}$

$$c(t) = 10\left[1 - e^{-t}\right] u(t)$$

b- $\left.\begin{array}{l} c(\infty) = 10 \\ c(0) = 0 \end{array}\right\} c(T) - 10 = -0.368 \times 10 = -3.68$

$$c(T) = 10 - 3.68 = 6.32$$

but $c(T) = 10 - 10e^{-T} = 6.32 \Rightarrow e^{-T} = 0.368$

$$\Rightarrow T = -\ln(0.368) = 0.999$$

c- The Time required for the step response to reach 63% of its final value

d- In this case, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{10}{s+1+10H}$

For $r(t) = u(t) \Rightarrow C(s) = \frac{10}{s[s+1+10H]} = \frac{10}{1+10H}\left[\frac{1}{s} - \frac{1}{s+1+10H}\right]$

$$c(t) = \frac{10}{1+10H}\left[1 - e^{-(1+10H)t}\right]u(t)$$

Based on (b) & (c),

$$c(T) - c(\infty) = 0.368\left[c(0) - c(\infty)\right], \quad c(0) = 0$$

$$c(\infty) = \frac{10}{1+10H}$$

$$c(0.5) = 0.63 \times \frac{10}{1+10H}$$

Substitute; we obtain $H = 0.988$
$\simeq 0.1$

Problem 6

$$G(s) = \frac{\frac{10}{3}(s+3)}{(s+1)(s+10)}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{10/3\,(s+3)}{s^2+\left(11+\frac{10}{3}\right)s+20} = \frac{10/3\,(s+3)}{s^2+(2\zeta\omega_n)s+\omega_n^2}$$

1- Comparing we obtain

$$\omega_n^2 = 20$$

$$2\zeta\omega_n = 11 + \frac{10}{3}$$

2- % overshoot $= e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.1 \Rightarrow \zeta = 0.91$

Substituting $\Rightarrow \frac{10}{\zeta} = 2(0,91)\sqrt{20} - 11 = -2,861$

$\Rightarrow \zeta = -3.49$

Problem 7

$G(s) = \frac{3s+10}{s^3+2s^2+s}$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{3s+10}{s^3+2s^2+4s+10}$$

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & 2 & \\ s^1 & -1 & \\ s^0 & 10 & \end{array}$$

System is unstable, Final Value Theorem cannot be applied

Problem 8

$$\frac{C(s)}{R(s)} = \frac{s+1}{s^6+s^5+5s^4+5s^3+7s^2+6s+3}$$

1.
$$\begin{array}{c|cccc} s^6 & 1 & 5 & 7 & 3 \\ s^5 & 1 & 5 & 6 & \\ s^4 & \cancel{0}\ \epsilon & 1 & 3 & \\ s^3 & \frac{5\epsilon-1}{\epsilon} & \frac{6\epsilon-3}{\epsilon} & & \\ s^2 & \frac{-6\epsilon^2+8\epsilon-1}{5\epsilon-1} & 3 & & \\ s^1 & \frac{36\epsilon^2+9\epsilon}{6\epsilon^2-8\epsilon+1} & & & \\ s^0 & 3 & & & \end{array}$$

2- $\epsilon > 0$

s^6	+
s^5	+
s^4	+
s^3	−
s^2	+
s^1	+
s^0	+

2 sign changes

2 RHP

4 LHP

3- $\epsilon < 0$

s^6	+
s^5	+
s^4	−
s^3	+
s^2	+
s^1	−
s^0	+

4 Sign Change

4 RHP

2 LHP.

4- The polynomial has roots on $j\omega$ axis. Replacing 0 by ϵ forces these roots to move either in the rhp or in the lhp.

5- $s = \frac{1}{p}$.

p^6	3	7	5	1
p^5	6	5	1	
p^4	9	9	2	
p^3	-1	$-1/3$		
p^2	6	2		
p^1	~~0~~ ~~1~~ 2			
p^0	2			

2 RHP

2 $j\omega$ axis

2 LHP.